Copper in Wind Power 2

Answers

1. What is the mathematical relationship between wind speed and power output?

Wind speed, or velocity, is the v in ½ mv^2 so you might think that the power depends on the square of v, but a little algebra will show that it depends on the cube of v.

The energy, E of the cylinder of air is given by:

$$E = \frac{1}{2} mv^2$$

As the air carrying this kinetic energy flows through in one second then this is also the power, so

$$P = \frac{1}{2} mv^2$$

Now the mass, m of air in the cylinder is given by the equation

$$m = \rho A l$$

where A is the swept area of the blades. But the air is travelling at l metres per second, which is the velocity V, so we can say

$$m = \rho A v$$

Putting this expression for m in the first equation gives

$$P = \frac{1}{2} \rho A v^3$$

This equation shows that the power of a turbine depends on the swept area and the wind speed. So it is best to build large turbines in windy places. This explains why offshore turbines are so effective.

2. What is the mathematical relationship between blade length and power output?

The blade length is the ‘r’ in πr^2, so the power output depends on the square of the blade length.

3. For the turbine in Figure 3, what is the maximum speed of the blade tips in km per hour?

Proportionally

At 11 rpm (revolutions per minute) calculate the tip speed in metres per second and kilometres per hour.

Circumference of tip circle = πd = 3.142 x 154m = 484m

At 11 rpm the distance travelled in one minute is 484 x 11 = 5321 m = 5.321 km

Converting to km per hour:

5.321 x 60 ≈ 320 kph

That is higher than the landing speed of the A380 airbus in the picture.

4. The equation for calculating electrical power is Power (in watts) = Volts x Amps (P=VI). If the generator is developing a voltage of 690 V and the power output is 6 MW, what is the current in amps running down the copper cables to the grid connection at the base of the tower?

Using the equation Power = Voltage x Current, calculate the current produced by the generator of a 6.0 MW turbine.

$P=VI$

P needs to be in Watts.

6.0 MW = 6×10^6 W

V = 690 V

So $I=P/V = 6 \times 10^6 / 690 \approx 8696$ A or 8.7 kA

Your domestic ring main is fused at 30 A.

8.7 kA (8700 A) is a very high current. The copper cables need to be very large to carry this current without overheating.

5. The tidal flow turbine uses the energy of flowing water. Find out the density of water and compare it to the density of air. How many times more kinetic energy is in flowing water compared to wind blowing at the same speed?

The density of air is 1.2 kg m^{-3} . The density of water is 1000 kg m^{-3}. The energy available for the same area and the same flow speed is 830 times higher than air. In reality, it is impossible to achieve that much extra, but moving water carries hundreds of times more energy than moving air flowing at the same speed.

These questions will guide you to the solution

What is the volume of air in the imaginary cylinder?

Volume = $\pi r^2 l$ = 3.142 x 15^2 x 15 m = 10,598 m^3

The density of air is 1.2 kg m^{-3}. What is the mass of the air?

Mass = ρV = 1.2 x 10,598 m^3 = 12,718 kg

If the length of the cylinder is 15 m and the wind speed is 15 ms^{-1} then the whole cylinder passes through the turbine in 1 second.

Kinetic energy = $1/2\, mv^2$. What is the kinetic energy of the air in the cylinder that passes through the turbine?

KE = $1/2\, mv^2$ = 1/2 x 12,718 x 15 x 15 = 2,862,788 J ≈ 1.4 MJ

If all that kinetic energy is transferred to the turbine in 1 s, what is the power of the turbine?

Power in Watts = Joules per second. Power = 1.4 MJs^{-1} = 1.4 MW

Do you think that all the energy will be transferred?

Even for a theoretically perfect turbine, it is impossible to take all the energy out of the air as it would stop and block the flow of more air. Betz's theory shows that you can only get 59% of the energy.

Even the best turbines can only get about 42% of the total wind energy. In our case this is just under 590 kW. This is about the right order of magnitude for that size of turbine, but still about three times the actual output.

Reasons for this include:

1. The centre part of the turbine contributes nothing to the turning force.
2. With three blades, some air simply passes through the gaps.
3. The generator is not 100% efficient. In fact electrical losses are tiny compared to aerodynamic losses.

If you are planning a wind farm, you need to know how much power is available for a given wind speed. Now you can go ahead and plan your farm.

Will altitude make any difference to available power?

The density of air drops with altitude, from about 1.2 kgm^{-3} at sea level to 0.85 kgm^{-3} at 3000 m altitude. However, wind speeds are usually higher at altitude and this offsets the loss. Turbines are more expensive to install and service at high locations.

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